Presentation Outline

- Flow through Pipes – Fundamentals
- Steady state analysis
- Hydraulic Transients
  - Terminology
  - Governing Equations
  - Numerical Solution
- Downstream valve closure
- Pressure wave propagation
Flow through pipes – Fundamentals

- Deals with fluid (mostly, water) flow in closed conduits
- Do not have a free surface (exert hydraulic pressure rather than atmospheric)
- Energy is expressed as ‘hydraulic head’ and expressed by ‘Bernoulli’s equation’
- ‘Friction’ causes the pressure drop or energy loss
- Major driving forces: Viscosity, pressure, etc.
- Major resisting force: Inertia (friction)
- Nature of flows: Laminar (Hagen-Poiseuille) and Turbulent (Darcy-Weisbach)
Flow through pipes – Fundamentals

• Circular pipes are used to withstand larger pressure differences between inside and outside (with no distortion)
• Fluid velocity inside the pipe changes from ‘zero’ at surface (no-slip) to maximum at centre

Problem Types in fluid flows

1) Determine pressure drop / head (given, pipe characteristics)
   Identify nature of flow; Use relevant flow equations

2) Determine flow rate (given, pipe characteristics and pressure drop)
   Guess friction factor, estimate flow rate, validate the guess value of ‘f’

3) Determine pipe diameter (design)
   Guess diameter, Calculate Re; pressure drop and compare with assumed diameter
Flow through pipes – Governing equations

**Continuity Equation**
- Based on conservation of mass (or, quantity) of fluid
  - Integral form:
    \[
    \frac{dq}{dt} + \oint_S \mathbf{j} \cdot dS = \Sigma
    \]
  - Differential form:
    \[
    \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0
    \]
- For, one-dimensional, steady-state case: \( Q = A_1 V_1 = A_2 V_2 \)

**Momentum Equation**
- Based on conservation of linear momentum
  \[
  \Sigma F_x = \rho_2 A_2 V_2 V_{2x} - \rho_1 A_1 V_1 V_{1x}
  \]

**Energy Equation**
- Based on conservation of energy in all forms
  \[
  \frac{P_1}{\rho} + g z_1 + \frac{V_1^2}{2} = \frac{P_2}{\rho} + g z_2 + \frac{V_2^2}{2} - W_p + W_t + W_f
  \]
Flow through pipes – HGL and EGL
Steady vs Un-steady Flows

**Steady state condition**
- The flow conditions (velocity, pressure and cross-section) may differ from point to point but do NOT change with time.

**Un-steady (transient) condition**
- At any point in the fluid, the flow conditions change with time.
- Transient condition can be ‘Gradual’ or ‘Rapid’.
- Rapid change in flow is more dangerous.
- If the average values are constant over time, the flow is considered steady.
Flow through pipes – Steady state

- In general, most of the pipe flow analysis is done considering steady-state approach
- Steady state flow assumption (or, condition) is meaningful when ...
  - There are no controls (valves) that result in change in velocity
  - There are no hydraulic machines that result in change in head/energy
  - Average flow conditions do not change with time
  - Flow velocity and pressure are low
  - The changes in head/energy are negligible

Applications of steady state flows
- Domestic water supply network
Hydraulic Transients – Prelude

Pipelines are designed to

• Operate under steady state condition
• Applies well-established and easy to understand principles

Minor Issues
• Actual flows may be slightly different from theoretical flows (aging, violation of assumptions, mis-interpretation, etc.)

Major Issues
• Cavitation and Un-steady flows
• Cavitation – During initial filling, high elevation, etc.
• Un-steady condition – Changes in internal velocity / pressure (valve operations, pump operations, power interruptions, etc.)

Bottom line: Pipelines are to be designed to withstand all possible transient conditions
Hydraulic Transients – Basics

Hydraulic Transient

- Any un-steady (time-varying) condition in the flow characteristics (velocity, pressure, etc)
- Preceded and succeeded by a steady-state / no-flow condition

Water Hammer

- A pressure surge or wave caused when fluid in motion is forced to stop
- Can result in noise and vibration followed by pipe collapse
- Analysis is done by considering compressibility of water and elasticity of pipe material (solves set of PDEs)

Surges

- Gradual transition in velocity or pressure
- Can damage pipe material and other parts
- Analysis is done using ‘rigid column theory’ (solves ODE)
Hydraulic Transients – Basics

Cause of hydraulic transients

- Change (open/closure) of valve
- Start / stop of pump
- Improper operation of check-valves, air-release valves, pressure-reducing valves, pressure relief valves, etc.
- Pipe rupture
- Trapped air in pipelines
- Changes in power demand of turbines,

Control of hydraulic transients

- Increase the closing / opening time of the valve
- Design special facilities for filling, flushing, and removing of air
- Increase pipe class; limit pipe velocity; reduce wave speed
- Use and maintenance of pressure relief valves, surge tanks, air chambers, etc.
Hydraulic Transients – Governing Equations

Continuity equation
\[
\frac{\partial H}{\partial t} + \frac{a^2}{gA} \cdot \frac{\partial Q}{\partial x} = 0
\]

Here, \( a = \) Velocity of pressure wave (so \( A = \) Cross-sectional area

Momentum equation
\[
\frac{\partial Q}{\partial t} + gA \frac{\partial H}{\partial x} + R \cdot \frac{\partial Q}{\partial x} = 0
\]

Here, \( R = f / 2DA \)

- These are coupled, non-linear, partial
- Theses equations are to be solved nur
Hydraulic Transients - Analysis

Numerical analysis of hydraulic transients can be done in TWO ways:

- Gradually varied condition (Lumped modeling)
  - Solves ODE (function of time only)
  - Fluid is assumed to be incompressible
  - Conduit walls are assumed to be rigid
  - Applicable when the valve closure is gradual

- Rapidly varied condition (Distributed modeling)
  - Solves PDE (function of space and time only)
  - Fluid is assumed to be slightly compressible
  - Conduit walls are assumed to be elastic
  - Applicable when the valve closure is rapid
Hydraulic Transients – MoC

On solving continuity and momentum equations through simplifications...

• The set of PDEs can be converted into OD
• Reduced equations (from MoC)

\[ Q_p = C_p - C_a \cdot H_p \quad \& \quad Q_p = C_n + \]

Solve the above equations for \( Q_p \) and \( H_p \).

Where,

\[
C_p = Q_L + C_a H_L - R Q_A |Q_A| \Delta t
\]

\[
C_n = Q_R - C_a H_R - R Q_B |Q_B| \Delta t
\]

\[
C_a = \frac{gA}{a}
\]

For numerical stability, \( \Delta X \geq a \cdot \Delta t \)

• The model domain (pipe) can be divided into segments (in X and t directions)
• Apply boundary conditions in length and time directions appropriately
Hydraulic Transients – Numerical solution

- Divide the pipe length into ‘n’ number of subdivisions, each of length ‘dx’
  
  *(at entrance, X_1 = 0 ; at valve / end, X_{n+1} = L)*

Specify ...

1) Initial steady state (t=0) conditions (along the x domain)
Means, specify Q and H, when t=0 at all nodes

2) Boundary conditions (when X = 0 or L)
Specify Q and H for all time steps along boundary nodes

Start from X=0, proceed in length direction (@ t=0)

Solve C+ and C- equations simultaneously, and get
  
  Q_p and H_p

Keep moving up in time direction
Hydraulic Transients – Boundary Conditions

- At up-stream Reservoir location
  
  - Point at u/s boundary is hit by a ‘C’ lir
    
    \[ H_{i}^{j+1} = H_R \]
    
    \[ Q_p = C_n + C_a \cdot H_R \]

- At down-stream valve location
  
  - Point at u/s boundary is hit by a ‘C’ lir
    
    \[ H_{n+1}^{j+1} = H_D + K_v \frac{V_{n+1}|V_{n+1}|}{2g} \]
    
    \[ Q_p = C_n + C_a \cdot H_R \]
Hydraulic Transients – Closure of a Valve

- When down-stream valve is ‘suddenly closed’
- At valve location, change in velocity, \( \Delta V = \)
- Change in pressure head, resulting from closure

- Wave celerity (pressure wave velocity):

  \[ K = \text{Bulk modulus for water} = 2.19 \times 10^5 \text{ Pa} \]
  \[ \rho = \text{Mass density of water} = 1000 \text{ kg/m}^3 \]
  \[ E = \text{Young's modulus of pipe material} = 2 \]
  \[ D = \text{Diameter of pipe material} \]
  \[ e = \text{Thickness of pipe walls} \]
Hydraulic Transients – Closure of a Valve

• Time required for the pressure wave to co
\[ \frac{L}{a} + \frac{L}{a} = \]

• Rapid closure of a valve: \( t_{cl} \ll \frac{2L}{a} \)

• Gradual closure of a valve: \( t_{cl} \gg \frac{2L}{a} \)
Hydraulic Transients – Closure of a Valve

1. $V = V_0$, $V = 0$
   
   $t = \varepsilon$

2. $V = 0$
   
   $t = \frac{L}{a}$

3. $V = -V_0$, $V = 0$
   
   $t = \frac{L}{a} + \varepsilon$

4. $V = -V_0$
   
   $t = \frac{2L}{a}$
Hydraulic Transients – Closure of a Valve

5. $V = -V_0$, $V = 0$
   \[ t = \frac{2L}{a} + \varepsilon \]

6. $V = 0$
   \[ t = \frac{3L}{a} \]

7. $V = V_0$, $V = 0$
   \[ t = \frac{3L}{a} + \varepsilon \]

8. $V = V_0$
   \[ t = \frac{4L}{a} \]
Hydraulic Transients – Pressure wave propagation

- **Case 1 & 2** (Propagation towards the reservoir)
  - On valve side, \( V = 0 \); piezometric head = \( H_0 + \Delta H \); Dia increases
  - On reservoir side, flow is un-disturbed; \( V_0 \); \( H_o \)

- **Case 3 & 4** (Propagation towards the valve)
  - Unstable equilibrium at the reservoir
  - On valve side, \( V = 0 \); piezometric head = \( H_0 + \Delta H \); Dia increases
  - On reservoir side, velocity = \(-V_0\); pressure = \( H_o \)

- **Case 5 & 6** (Propagation towards the reservoir)
  - Unstable equilibrium at the valve
  - On valve side, \( V = 0 \); piezometric head = \( H_0 - \Delta H \)
  - On reservoir side, flow is un-disturbed; \(-V_0\); \( H_o \)

- **Case 7 & 8** (Propagation towards the valve)
  - Unstable equilibrium at the reservoir
  - On valve side, \( V = 0 \); piezometric head = \( H_0 - \Delta H \)
  - On reservoir side, velocity = \( V_0 \); pressure = \( H_0 \)
Hydraulic Transients – Pressure wave propagation

Pressure variation at valve: velocity head and friction losses neglected

Neglecting head loss!
Pressure variation at valve: velocity head and friction losses are considered
Thank you